COMPARATIVE STUDY IN THE LEVELING NETWORKS SOLVING

Arsene[1] C., M.V. Ortelecan[2]

[1]*Technical University of Cluj-Napoca, Romania; cornelarsen@yahoo.com*
[2] *University of Agricultural Sciences and Veterinary Medicine Cluj-Napoca, Romania*

Abstract. The paper presents the way of approach in a levelling network solving and how to choose the right process, depending on the complexity of the network so that the imposed precision requirements are fulfilled.

Keywords: Leveling networks, solution processes

INTRODUCTION

When you need to realize calculations in order to solve a geodetic network-in this case a leveling network, the computing method chosen should ensure the obtaining of results matching the degree of accuracy required for such papers and, in the same time, does not require a large volume of calculations, even though for the time being, due to the automation of the computing processes, high volume is no longer an impediment.

The solving method chosen is strongly related to the network`s configuration and complexity, and the method of calculation itself must allow the automation of the papers in a much bigger measure.

MATERIAL AND METHOD

Generally the observations in case of leveling networks are realized by means of geometric leveling and rarely by means of trigonometric leveling.

For the calculations, the most widely used methods are: the conditional measurements method and the indirect measurements method.

A particularity of the conditional measurements method, also called the method of the polygons in case of leveling networks, is the fact that the measured quantities must satisfy certain conditions caused by the network`s configuration; these conditions create the conditioned equations, wich, after solving the calculations, help you determine the corrections of the measured quantities.

For the indirect measurements method, each size measured will generate a correction equation so that the system of correction equations is composed by a number of equations that equals the number of measured quantities.

The leveling network under study has a total of three polygons formed by eight leveling lines. (Fig. 1.)

The observations were made with the method geometric middle leveling, using precision tools and special stages of invar.

Effective processing of the observations can be made either by the process of gradual reductions, or by the matrix process, the last one managing the automation better.
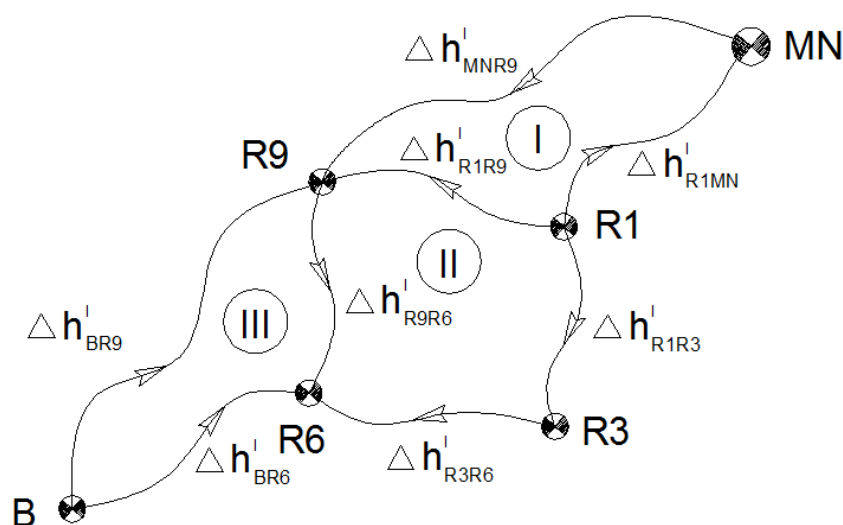
Fig. 1. The leveling network

The dates obtained after performing the observations and the preliminary calculations can be found in the first Table 1.

Table 1

Linia de nivelment	Dif. nivel mas. $\Box h'_{ij}$ [m]	Lung. liniei L [km]	Cote proviz. H'_i [m]	Cote defin. H_i [m]	Punct
R1-MN	0.1510	0.172782		340.30300	MN
R1- R3	0.1472	0.060321	340.15430		R1
R3-R6	1.7864	0.034232	340.30130		R3
B-R6	0.0991	0.068423	342.08860		R6
B-R9	0.0962	0.143829	341.98920		B
MN-R9	1.7830	0.299699	342.08480		R9
R1-R9	1.9312	0.223279			
R9-R6	0.0040	0.061806			

RESULTS AND DISCUSSION

In the case of solving the leveling network using the conditioned measurements method, the number of equations by r condition is given by the sum of the real polygons P_r and the number of fictional polygons P_f of the network:

$$r = P_r + P_f \tag{1}$$

the number of fictional polygons is established with the relation:

$$P_f = P_v - 1 \tag{2}$$

P_v − the number of known points of the leveling network.

Starting from the condition that the sum of level differences in a polygon equals zero, the liniarized form of the condition equations, for the network considered, is:

$$\begin{cases} -v_{R1MN} & -v_{MNR9} + v_{R1R9} & +\omega_I = 0 \\ \quad v_{R1R3} + v_{R3R6} & -v_{R1R9} - v_{R9R6} + \omega_{II} = 0 \\ \quad -v_{BR6} + v_{BR9} & +v_{R9R6} + \omega_{III} = 0 \end{cases} \tag{3}$$

The free terms ω are obtained so:

$$\omega_I = -\Delta h'_{R1MN} - \Delta h'_{MNR9} + \Delta h'_{R1R9}$$
$$\omega_{II} = \Delta h'_{R1R3} + \Delta h'_{R3R6} - \Delta h'_{R1R9} - \Delta h'_{R9R6}$$
$$\text{(4)}$$
$$\omega_{III} = -\Delta h'_{BR6} + \Delta h'_{BR9} - \Delta h'_{R9R6}$$

The system of normal equations corresponding the system of condition equations (3), is:

$$\begin{cases} P_I\, k_I - L_{R1R9} k_{II} & +\omega_I = 0 \\ -L_{R1R9}\, k_I + P_{II}\, k_{II} - L_{R9R6} k_{III} + \omega_{II} = 0 \\ -L_{R9R6}\, k_{II} + P_{III}\, k_{III} + \omega_{III} = 0 \end{cases} \tag{5}$$

Coefficients L_{ij} reprezent the length of theleveling line ij (expressed in km), while the coefficients P_i of the correlates from the main diagonal wich reprezent the length of the polygon i (in km).

To facilitate calculations, solving the system of equations is done by the matrix process.

The matrix relationship corresponding is:

$$v = QA(A^T QA)^{-1}\omega = QAk \tag{6}$$

In the expression (6):

v - the column matrix of the corrections,

A - the multipliers matrix of the unknowns;

A^T - the A`s matrix transpose;

Q - the matrix of the leveling lines weights;

ω - the column matrix of the free terms;

k - the corelated column matrix.

Numerically, the matrixes have the form:

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad \omega = \begin{pmatrix} 2.8 \\ 1.6 \\ -1.1 \end{pmatrix}$$

(7)

$$Q = \begin{pmatrix} 0.172782 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.223279 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.061806 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.068423 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.143829 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.299699 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.060321 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.034232 \end{pmatrix}$$

By solving the calculation the column matrix of the correlated re obtained, respectively the corrections:

$$k = \begin{pmatrix} 6.480494 \\ 7.653511 \\ -2.287717 \end{pmatrix} \qquad v = \begin{pmatrix} -1.119713 \\ -0.261910 \\ -0.614428 \\ 0.156532 \\ -0.329040 \\ -1.942197 \\ 0.461667 \\ 0.261995 \end{pmatrix} \text{ [mm]} \qquad (8)$$

The values of the corrected differences in level Δh_{ij} and offset quotations H_i are reproduced in Table 2

Table 2

Linia de nivelment	Dif. nivel mas. $\Box h'_{ij}$ [m]	Cote proviz. H_i [m]	Corectii v_{ij} [m]	Dif. nivel corect. Δh_{ij} [m]	Cote compens. H_i [m]	Punct
R1-MN	0.1510		-0.001120	0.149880	340.303000	MN
R1-R9	1.9312	340.1543	-0.000262	1.930938	340.153120	R1
R9-R6	0.0040	342.0848	-0.000614	0.003386	342.084058	R9
B-R6	0.0991	342.0886	0.000157	0.099257	342.087443	R6
B-R9	0.0962	341.9892	-0.000329	0.095871	341.988187	B
MN-R9	1.7830	340.3013	-0.001942	1.781058	340.300781	R3
R1-R3	0.1472		0.000462	0.147662		
R3-R6	1.7864		0.000262	1.786662		

When for the leveling network solving, is used the indirect measurements method, the number of correction equations will be equal with the number of measured sizes, namely the number of lines of leveling network.

The form of the correction equations is the points nature which border the leveling line considered, respectively if the leveling line is between an old point and a new one, or if it is between two new points.

The correction equations system for the leveling network in Fig. 1. has the form:

$$
\left\{
\begin{aligned}
v_{R1MN} &= -dx_{R1} & -2.3 \quad ; p_{R1MN} &= 0.578764 \\
v_{R1R3} &= -dx_{R1} + dx_{R3} & -0.2 \quad ; p_{R1R3} &= 1.657797 \\
v_{R3R6} &= -dx_{R3} + dx_{R6} & +0.9 \quad ; p_{R3R6} &= 2.921243 \\
v_{BR6} &= dx_{R6} - dx_B & +0.3 \quad ; p_{BR6} &= 1.461497 \\
v_{BR9} &= -dx_B + dx_{R9} & -0.6 \quad ; p_{BR9} &= 0.695270 \\
v_{MNR9} &= dx_{R9} & -1.2 \quad ; p_{MNR9} &= 0.333668 \\
v_{R1R9} &= -dx_{R1} + dx_{R9} & -0.7 \quad ; p_{R1R9} &= 0.447870 \\
v_{R9R6} &= dx_{R6} - dx_{R9} & -0.2 \quad ; p_{R9R6} &= 1.617966
\end{aligned}
\right. \tag{9}
$$

Free factors l_{ij} of the system (9) are determined by the relations:

$$
\begin{aligned}
l_{R1MN} &= H'_{R1}-\left(H_{MN} + \Delta h'_{R1MN}\right) \\
l_{R1R3} &= H'_{R3}-\left(H'_{R1}+\Delta h'_{R1R3}\right) \\
l_{R3R6} &= H'_{R6}-\left(H'_{R3}+\Delta h'_{R3R6}\right) \\
l_{BR6} &= H'_{B}-\left(H_{R6} + \Delta h'_{BR6}\right) \\
l_{BR9} &= H'_{R9}-\left(H'_{B}+\Delta h'_{BR9}\right) \\
l_{MR9} &= H'_{R9}-\left(H_{MN} + \Delta h'_{MNR9}\right) \\
l_{R1R9} &= H'_{R9}-\left(H'_{R1}+\Delta h'_{R1R9}\right)
\end{aligned} \tag{10}
$$

$$l_{R9R6} = H'_{R6} - \left(H'_{R9} + \Delta h'_{R9R6} \right),$$

while the weights p_{ij} are determined by the relations:

$$p_{ij} = \frac{c}{L_{ij}} \qquad\qquad p_{ij} = \frac{c}{S_{ij}} \tag{11}$$

L_{ij} – Leveling line length [km];

S_{ij} – Number of stations on the leveling line.

By normalizing the error equations in the system of equations (9) they become:

$$\left(p_{R1MN} + p_{R1R3} + p_{R1R9} \right) dx_{R1} - p_{R1R3}\, dx_{R3} - p_{R1R9}\, dx_{R9} +$$
$$+ \left(- p_{R1MN} l_{R1MN} - p_{R1R3}\, l_{R1R3} - p_{R1R9}\, l_{R1R9} \; \right) = 0$$

or:

$$P_{R1}\, dx_{R1} - p_{R1R3}\, dx_{R3} - p_{R1R9}\, dx_{R9} + E_{R1} = 0 \tag{12}$$

As a result, for the leveling network considered, the system of normal equations has the form:

$$\begin{cases} P_{R1}\, dx_{R1} - p_{R1R3}\, dx_{R3} \qquad\qquad\qquad\qquad - p_{R1R9}\, dx_{R9} + E_{R1} = 0 \\ - p_{R1R3}\, dx_{R1} + \;\; P_{R3}\, dx_{R3} - p_{R3R6}\, dx_{R6} \qquad\qquad\qquad\quad + E_{R3} = 0 \\ \qquad\qquad - p_{R3R6}\, dx_{R3} + \;\; P_{R6}\, dx_{R6} - p_{BR6}\, dx_{B} - p_{R9R6}\, dx_{R9} + E_{R6} = 0 \\ \qquad\qquad\qquad\qquad - p_{BR6}\, dx_{R6} + \;\; P_{B}\, dx_{B} - \;\; p_{BR9}\, dx_{R9} \;\; + E_{B} = 0 \\ - p_{R1R9}\, dx_{R1} \qquad\qquad\qquad - p_{R9R6}\, dx_{R6} - p_{BR9}\, dx_{B} + \;\; P_{R9}\, dx_{R9} \;\; + E_{R9} = 0 \end{cases} \tag{13}$$

The corresponding value of the normal equations system (13) is:

$$\begin{cases} 2.684432\, dx_{R1} - 1.657797\, dx_{R3} \qquad\qquad\qquad\quad - 0.447870\, dx_{R9} + 1.976226 = 0 \\ - 1.657797\, dx_{R1} + 4.579041\, dx_{R3} - 2.921243\, dx_{R6} \qquad\qquad\qquad - 2.960678 = 0 \\ - 2.921243\, dx_{R3} + 6.000706\, dx_{R6} - 1.461497\, dx_{B} - 1.617966\, dx_{R9} + 2.743975 = 0 \\ \qquad\qquad\qquad - 1.461497\, dx_{R6} + 2.156767\, dx_{B} - 0.695270\, dx_{R9} - 0.021287 = 0 \end{cases}$$
(13')

$$- 0.447870\, dx_{R1} - 1.617966\, dx_{R6} - 0.695270\, dx_{B} + 3.094774\, dx_{R9} - 0.807480 = 0$$

In the relations (13') the weights P_i were calculated like this:

$$P_{R1} = p_{R1MN} + p_{R1R3} + p_{R1R9}$$

$$P_{R3} = p_{R1R3} + p_{R3R6}$$
$$P_{R6} = p_{R3R6} + p_{BR6} + p_{R9R6}$$
$$P_B = p_{BR6} + p_{BR9} \tag{14}$$
$$P_{R9} = p_{MNR9} + p_{R1R9} + p_{BR9} + p_{R9R6},$$

and the free terms E_i with the expressions:

$$E_{R1} = -p_{R1MN}l_{R1MN} - p_{R1R3}\,l_{R1R3} - p_{R1R9}\,l_{R1R9}$$
$$E_{R3} = p_{R1R3}l_{R1R3} - p_{R3R6}l_{R3R6}$$
$$E_{R6} = p_{R3R6}l_{R3R6} + p_{BR6}l_{BR6} + p_{R9R6}l_{R9R6} \tag{15}$$
$$E_B = -p_{BR6}l_{BR6} - p_{BR9}l_{BR9}$$
$$E_{R9} = p_{MNR9}l_{MNR9} + p_{R1R9}l_{R1R9} + p_{BR9}l_{BR9} - p_{R9R6}l_{R9R6}$$

The matrix form of the normal equations system is:

$$A^T PAx + A^T PE = 0 \tag{16}$$

In the expression (16):

A - the matrix of the unknown multipliers;
A^T - the transpose of the A matrix;
P - the matrix of the leveling network`s weights;
E - the column matrix of the free terms;
x - the column matrix of the unknown dx_i.

Using the notations:

$$A^T PA = N \;\; ; \;\; A^T PE = E^* \tag{17}$$

the relation (1-16) becomes:

$$Nx + E^* = 0 \tag{18}$$

By solving the normal system (18), for the unknown dx_i is obtained the solution:

$$x = -N^{-1}E^* \qquad x = \begin{pmatrix} -1.180287 \\ -0.518620 \\ -1.156625 \\ -1.013157 \\ -0.742197 \end{pmatrix} \text{[mm]} \qquad (19)$$

The matrix of the unknown cofactors Q_x is:

$$Q_x = N^{-1} \qquad Q_x = \begin{pmatrix} 0.633333 & 0.533333 & 0.433333 & 0.4 & 0.366667 \\ 0.533333 & 1.133333 & 0.733333 & 0.6 & 0.466667 \\ 0.433333 & 0.733333 & 1.033333 & 0.8 & 0.566667 \\ 0.4 & 0.6 & 0.8 & 1.2 & 0.6 \\ 0.366667 & 0.466667 & 0.566667 & 0.6 & 0.633333 \end{pmatrix} \qquad (20)$$

The compensated values of the H_i parameters, also the ones of the measured Δh_{ij} elements, are established with the relations:

$$H_i = H'_i + dx_i \qquad (21)$$
$$\Delta h_{ij} = \Delta h'_{ij} + v_{ij} \qquad (22)$$

the corrections v_{ij} are given with the formulae:

$$v = Ax + l \qquad (23)$$

The numerical values of the compensated quotations and differences of level ($H_i, \Delta h_{ij}$), and also of the corrections v_{ij} are presented in the Table 3.

Table 3

Linia de nivelment	Dif. nivel mas. $\Box h'_{ij}$ [m]	Corectii v_{ij} [m]	Dif. nivel corect. Δh_{ij} [m]	Cote proviz. H'_i [m]	Necunosc dx_i [m]	Cote compens. H_i [m]	Punct
R1-MN	0.1510	-0.001120	0.149880	340.3030		340.303000	MN
R1-R3	0.1472	0.000462	0.147662	340.1543	-0.001180	340.153120	R1
R3-R6	1.7864	0.000262	1.786662	340.3013	-0.000519	340.300781	R3
B-R6	0.0991	0.000157	0.099257	342.0886	-0.001157	342.087443	R6
B-R9	0.0962	-0.000329	0.095871	341.9892	-0.001013	341.988187	B
MN-R9	1.7830	-0.001942	1.781058	342.0848	-0.000742	342.084058	R9
R1-R9	1.9312	-0.000262	1.930938				
R9-R6	0.0040	-0.000614	0.003386				

The precision of determination the corrections v_{ij} and the unknown dx_i is obtained by calculating the standard aberrance of the weight digit s_0 and the standard aberrance of the unknown dx_i marked with s_{xi}:

$$s_0 = \sqrt{\frac{[pvv]}{n-k}} = 1.0473377 \text{ mm/km} \tag{24}$$

$$s_{xi} = s_0 \sqrt{Q_{xkxk}} \qquad\qquad s_{xi} = \begin{pmatrix} 0.833495 \\ 1.114976 \\ 1.064650 \\ 1.147301 \\ 0.833495 \end{pmatrix} \text{ [mm]} \tag{25}$$

CONCLUSIONS

Analyzing the obtained results, it appears that they are the same regardless of the method used to solve.

At the same time it is noted that, for the considered leveling network, in the case of the subject measurements method, the original system of equations is formed out of three equations, one for each polygon, while in the case of the indirect measurements method, the number of equations is equal with the number of leveling lines – which means eight equations, nearly three times higher than the subject measurements method.

Thus, in case of the surveying networks where the number of r conditioned equations are less than the number of new points in the network, for solving, the subject measurements method involves a greatly reduced working volume that provides the same results, however, as well as indirect measurement method.

But when the leveling network has a complex configuration, the indirect measurements method provides an easy writing system of the correction equations, each measured variable generating an equation is ruled out the possibility of wrong writing the system of equations, which can occur when the conditional measurement method is used.

REFERENCES

1. Arsene C., Ortelecan M.V. (2010), The pursuit of behavior through time of some constructions with the structure made of concrete and masonry, Agricultura no. 3–4 (75-76)/2010.

2. Arsene C., Ortelecan M.V. (2010), The behavior through time of a metallic bay under the effect of moistness of the earthwork soil, Agricultura no. 3–4 (75-76)/2010.

3. Ghiţău D. (1983), Geodezie şi gravimetrie geodezică, Ed. Didactică şi Pedagogică, Bucureşti.