Aspects Regarding the Establishment of the Scale Coefficient in the Case of Distances Measurements in a Geodetic Network

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Abstract

The paper analyzes the possibility to establish the coefficient of scale towards the total station scale triangulation network in the conduct of geodetic and topographic observations in the points with known coordinates (old points) or points whose coordinates we want to determine (new points). The purpose of the study is undertaken to simplify computing operations to reduce distances measured from the topographic surface to the Stereo 70 projection plan.

Keywords: *geodetic network, linear intersections, scale coefficient*

INTRODUCTION

A geodetic network, achieved in a particular projection system is characterized by a coefficient of scale given by the coordinates of the old points.

To calculate the coordinates of the points of the geodetic network, the measurable elements (azimuthal directions, distances and zenith) are projected on the reference surface (ellipsoid of revolution) and then used in the project plan.

In the case of solving the geodetic networks using the trilateration method, reducing the distances measured on the ellipsoid Krasovski and then Stereographic 1970 projection plan is mandatory.

Using the functional- stochastic model, the distances measured on the topographic surface are corrected with the following corrections:

- Reducing the distances at chord on ellipsoid (D');
- Reducing on the reference ellipsoid surface (s);
- Reducing in the Stereographic 1970 projection plan.

Such relationships are used:

$$D' = \sqrt{\frac{D_s^2 - \Delta h_{12}^2}{\left(1 + \frac{H_1^E}{R_m}\right)\left(1 + \frac{H_2^E}{R_m}\right)}} \quad (1)$$

where:

D_s – slope distance between points 1 and 2, after eliminating the physiques corrections;

D' – the distance reduced to the chord on the ellipsoid or on the sphere of medium radius;

s – the distance reduced on the reference surface (geodesic line);

R_m – the medium radius;

H_1^E, H_2^E – ellipsoidal level heights;

Dh_{12}^E – the ellipsoidal level difference between points 1 and 2.

$$s = 2R_m \frac{\omega}{2} = 2R_m \arcsin \frac{D'}{2R_m} \quad (2)$$

Fig.1. Reducing the distances on the reference surface

$$R_m = \sqrt{MN} \quad (3)$$

where:

M – the radius of the meridian ellipse;

N – the radius of the first vertical;

B_m – the medium latitude of the measured length.

$$M = \frac{a(1-e^2)}{\sqrt{(1-e^2 \sin^2 B_m)^3}} ;$$

$$N = \frac{a}{\sqrt{(1-e^2 \sin^2 B_m)}} \quad (4)$$

The quantities which are calculated for our country, depending on the Krasovski ellipsoid parameters:

a – major semi-axis ($a = 6378245$ m);

f – ellipsoid flattening ($f = 1/198.3$);

e – first excentricity ($e^2 = (a^2 - b^2)/a^2$)

$$\mu = 1 + \frac{s^2}{4R_0^2} - \frac{1}{4000} = 0.999750 + \frac{x_m^2 + y_m^2}{4R_0^2} \quad (5)$$

m – represents the lengths deformation module, at the crossing from the ellipsoid in the secant plane of the stereographical 1970 projection.

In the case of geodetic network development using the multiple linear intersections method, the measuring instrument scale is different from the network scale and in this case requires that the measured values to be brought to the scale of the geodetic network by entering a scale factor.

Although in the specialty literature is noted that between the old points of geodetic network are not performed distances measurements (Moldoveanu, 2002; Atudorei, 1981), for determining the scale coefficient of the total station and of the network, in this paper, we will also study the measured distances between old points.

MATERIALS AND METHODS

The study was achieved on the east part of Cluj-Napoca city (fig.2), on the triangulation points 250, 252, 253, given by the Stereo 1970 coordinates (Tab.1)

The azimuth, zenith and distances measurements were achieved with Leica TCR 805 total station and round prisms, positioned vertically using metal tripods. The mean values of the measurements are shown in Table 2

In table 2, the notations have the following significations:

Hz – azimuth directions;

V – zenith directions;

SD – slope distances;

HD – horizontal distances.

]

Tab. 1.

Pct.	X [m]	Y [m]	H ^N [m]	H ^E [m]
250	586099.150	399284.500	424.510	465.128
253	586811.570	400084.260	361.140	401.719
252	585419.130	400957.880	438.190	478.766

Tab. 2.

Point		Hz	V	SD	HD
St	Vi	[g.c.cc.]	[g.c.cc.]	[m]	[m]
250	253	53.6701	103.7679	1073.187	1071.318
	252	124.5685	99.5252	1806.720	1806.666
253	252	164.3331	97.0247	1645.954	1644.138
	250	253.6735	96.2352	1073.198	1071.312

Fig.2. Position of the points taken into study, represented on the plan L-34-48-C-b **Fig.3.** The triangulation points taken into study

Tab. 3.

Distance name	Value [m]
d _{250,253}	1071.055
d _{253,252}	1643.807
d _{250,252}	1806.275

Tab. 4.

Distance name	Calculated distance [m]	Measured distance [m]	C	C ₁
d _{250,253}	1071.055	1071.318	0.999754	1.000246
d _{253,252}	1643.807	1644.138	0.999799	1.000201
d _{250,252}	1806.275	1806.666	0.999783	1.000217

RESULTS AND DISCUSSIONS

From the coordinates presented in table 1, it will be calculated the distances from the Stereo 1970 projection plan, applying the relation:

$$d_{ij} = \frac{\Delta x_{ij}}{\cos \theta_{ij}} = \frac{\Delta y_{ij}}{\cos \theta_{ij}} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2} \quad (6)$$

The values of the distances calculated from the points coordinates, for the three sides, are presented in table 3.

Given that the difference between horizontal distance and distance measured on the surface from reference projects is of the order of millimeters, for lengths shorter than 10 km, the scale coefficient will be calculated as a ratio between the distance calculated from coordinates and the horizontal distance measured.

$$C = \frac{d_{ij}}{D_{ij}} \quad (7)$$

where:

d_{ij} – distance calculated from coordinates;
D_{ij} – measured distance.

The coefficient of recovery from network scale to the scale of the total station is calculated with the following relation:

$$C_1 = \frac{1}{C} \quad (8)$$

The calculated values of the scale coefficient of the network and of the total station, for the three sides, are presented in table 4.

In order to check the value of the scale factor of the total station, it is allowed the assumption that the point 250 is a new point from which polar measurements were made at two points of known coordinates, 253 and 252. Depending on the measured elements, given in table 2, it will be

calculated the linear horizontal length of 253-252 base, with the relationship:

$$D_{253,252}^2 = D_{250,253}^2 + D_{250,252}^2 - 2D_{250,253}D_{250,252} \cos \omega = 1644.143 \text{ m} \quad (9)$$

where:

$$\omega = H_{z_{252}} - H_{z_{253}} = 124.5685 - 53.6701 = 70.8984^\circ \quad (10)$$

The scale coefficient is calculated with relation (7), using the values determined by the relations (6) and (9):

$$C = \frac{d_{253,252}}{D_{253,252}} = \frac{1643.8069}{1644.143} = 0.9997955 \quad (11)$$

$$C_1 = \frac{1}{C} = \frac{1}{0.9997955} = 1.0002045 \quad (12)$$

The difference between the two determinations of the scale module is of 3.41259E-06.

To calculate the provisional coordinates of point 250, assuming that this is a new point the orientation $\theta_{253,250}$ is calculated with the relationship:

$$\theta_{253,250} = \theta_{253,252} + \omega_1 = 164.3286 + 89.3446 = 253.6732^\circ \quad (13)$$

in which:

$$\theta_{253,252} = \arctg \frac{\Delta y_{253,252}}{\Delta x_{253,252}} = 164.3286^\circ \quad (14)$$

$$\omega_1 = \arccos \left(\frac{(C_1 d_{253,252})^2 + D_{253,250}^2 - D_{253,250}^2}{2C_1 d_{253,252} D_{253,250}} \right) = 89.3446^\circ \quad (15)$$

The provisional coordinates of point 250 are calculated with the relations:

Tab. 5.

Pct.	Known coordinates		Calculated coordinates		Accuracy	
	X [m]	Y [m]	X [m]	Y [m]	Dx [m]	Dy [m]
250	586099.150	399284.500	586099.129	399284.468	0.021	0.032

Tab. 6.

Pct.	Known coordinates		Calculated coordinates		Accuracy	
	X [m]	Y [m]	X [m]	Y [m]	Dx [m]	Dy [m]
253	586811.570	400084.260	586811.632	400084.292	-0.062	-0.032
253*	586811.570	400084.260	586811.574	400084.329	-0.004	-0.069

$$\begin{aligned}
 X_{250} &= X_{253} + C * D_{253,250} * \cos \theta_{253,250} = 586811.570 - 712.441 = 586099.129 \text{ m} \\
 Y_{250} &= X_{253} + C * D_{253,250} * \sin \theta_{253,250} = 400084.260 - 799.792 = 399284.468 \text{ m}
 \end{aligned}
 \quad (16)$$

By calculating the difference between the known coordinates of the point taken into study and the values calculated, under the assumption that the point is new, it obtains the accuracy of positioning point (Table 5)

Similarly, accepting the hypothesis that point 253 is a new point will be calculated the linear 250-252 base length, measured from the point elements considered new. Calculations will be performed identically shaped relationship (9) , (16).

$$D_{250,252}^2 = D_{253,252}^2 + D_{253,250}^2 - 2D_{253,252}D_{253,250} \cos \omega_1 = 1806.5985 \text{ m} \quad (17)$$

$$\omega_1 = H_{z_{250}} - H_{z_{252}} = 253.6735 - 164.3331 = 89.3405^\circ \quad (18)$$

$$C = \frac{d_{250,252}}{D_{250,252}} = \frac{1806.275}{1806.5985} = 0.999820711 \quad (19)$$

$$C_1 = \frac{1}{C} = \frac{1}{0.999820711} = 1.0001793 \quad (20)$$

$$\theta_{252,253} = \theta_{252,250} + \omega = 324.5729 + 39.7580 = 364.3309^\circ \quad (21)$$

$$\theta_{252,250} = \arctan \frac{\Delta y_{252,250}}{\Delta x_{252,250}} = 324.5729^\circ \quad (22)$$

$$\omega_2 = \arccos \left(\frac{(C_1 d_{253,252})^2 + D_{253,250}^2 - D_{250,252}^2}{2C_1 d_{253,252} D_{253,250}} \right) = 39.7580^\circ \quad (23)$$

$$\begin{aligned}
 X_{253} &= X_{252} + C * D_{253,252} * \cos \theta_{252,253} = 585419.130 + 1392.502 = 586811.632 \text{ m} \\
 Y_{253} &= X_{252} + C * D_{253,252} * \sin \theta_{252,253} = 400957.880 - 873.588 = 400084.292 \text{ m}
 \end{aligned}
 \quad (24)$$

The accuracy of the provisional coordinates of point 253 established, compared to the known points is presented in table 6.

Provisional coordinate values of point 253 (shown as point 253*) calculated with the medium scale coefficient obtained by the values shown in Table 4, are also presented in Table 6.

Our calculations with relation (7) and (11) showed that the difference between the scale module side is 3.41259E-06 for the side 253-252 and -3.72108E-05 for the side 250-252.

The accuracy of provisional positioning of point 250, assuming that it is a new point, is of 0.0385 m and of the point 253 is 0.070 m.

New points coordinate values fall within the tolerances allowed against the same coordinates points obtained by the classical method of designing distances reference surface and then used in the project plan.

CONCLUSIONS

Using the scale coefficient, through which the measured elements are brought to the network scale, eases a lot the stages of computing of the new points coordinates.

To increase positioning accuracy in each new point must converge at least three sides, measured in points with known coordinates or calculated in the adopted reference system.

At least two of the sides that converge in each new point, must form between them an angle close to the right angle.

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