FEATURES OF THE BERTALANFFY-RICHARDS GROWTH MODEL IN FORESTRY

Aldea Florica, Ioana Pop, Maria Micula, M. Dirja, V. Budiu

Faculty of Horticulture, USAMV Cluj-Napoca, Calea Manastur 3-5, Cluj-Napoca, Romania, faldea@usamvcluj.ro

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Abstract. Due to the importance of the forest to the environment there is a lot of interest for the growth model of the forest and there are many growth model such as: logistic, the Gompertz, the Schmitte etc. In this paper we have studied the Bertalanffy-Richards growth model and its features.

INTRODUCTION

One of the major problems of the environment is the landslide and the forest plays an important role in this problem. In our country in the latest years had place massif deforestation. One possible solution of this is the forestation and for the good results are necessary studies of the growth for the forest. We start to study this issue in the paper [1].

Soil erosion and landslide are some of the prevalent land degradation process on Romania. The land with slope greater than 5% represents 67% from the territory of Romania. The fast soil erosion is favoured by the geomorphologic and litological conditions. The decreasing of the areas covered by forest represents the most important factor for the soil degradation. The surface covered by forest represents 27% from the territory of Romania.

Developing mathematical models has become an integral part of the research in many areas of forestry, natural resources and environment sciences. There are cases where is physically impossible to conduct certain experiments. Experiments of these types must be conducted through mathematical models and the result can be used to test the hypothesis of interest. Mathematical models can also be used to generate information necessary for making appropriate decisions [3.].

The major difference between the model presented in paper [1.] and the Bertalanffy-Richards growth model consists in the nonlinearity. Due to this nonlinearity some features are better emphasised for this model [2.].

MATHERAL AND METHODE

An important role for the growth models it plays by the shape of the growth function. For this reason Bertalanffy-Richards model stat from the classical model

\[ y' = k(\alpha - y) \]  \hspace{1cm} (1)

where \( y = y(t) \) is the growth function, \( \alpha \) and \( k \) are constants [1]. Much greater flexibility is obtained by substituting \( y \) by \( y' \). From the equation (1) and previous substitution we obtained

\[ y' = \eta y'' - ry \]  \hspace{1cm} (2)
where $\eta = \frac{k\alpha^\gamma}{v}$, $m = 1 - v$, $r = k / v$. Equation (2) is the Bertalanffy-Richards growth model.

Solution of equation (2) depends on the initial condition for the differential equation. If that growth model illustrates the forest growth, $y(t)$ represents the number of trees at moment $t$ and is obtained as the solution of differential equation (2). For our study, we assume that at the initial moment $t = 0$ the number of trees are $y_0$. The coefficient $\eta$ and $r$ contain information about soil and climacteric conditions, species involved and human factor.

RESULT AND DISCUSSIONES

The shape of the growth function is given by the solution of Cauchy problem

\[
\begin{aligned}
\left\{
\begin{array}{l}
y' = \eta y^m - ry \\
y(0) = y_0
\end{array}
\right.
\end{aligned}
\]  \tag{3}

The solution for the Cauchy problem (3) is

\[
y(t) = y_0 \left( \frac{r}{\eta y_0^{m-1} + (r - \eta y_0^{m-1})e^{r(m-1)t}} \right)^{1/m-1}
\]  \tag{4}

The features for the solution of Cauchy problem (4) are given below together with the discussions about parameters $m$, $\eta$ and $r$ [4.].

1. $m > 1, \eta < 0, r < 0$
   In this case function (4) has the following form:

\[
y(t) = A(1 - Be^{-kt})^{1/(1-m)}
\]  \tag{5}

where:

- $A = (\eta / r)^{1/(1-m)}$ is the asymptote value of the response $y$;
- $B = (1 / r - y_0^{1-m}) / (\eta / r)$ is the biological constant;
- $k$ is proportional with $y$;
- $m$ is the allometric constant that means it is responsible with shape of growth function.

Fig. 1 The shape of growth function when $m > 1, \eta < 0, r < 0$
II. $0 < m < 1, \eta > 0, r > 0$
Using the same constant as in previous item we have the following shape for the growth function

![Fig. 2 The shape of growth function when $0 < m < 1, \eta > 0, r > 0$](image)

III. $m < 0, \eta > 0, r < 0, r' = -r$

The growth function (4) has the following form:

$$y(t) = D(\eta e^{kr} + 1)^{1/(1-m)}$$  \hspace{1cm} (6)

where

$$C = (\frac{\eta}{r'})^{1/(1-m)}$$

$$E = 1 + \left(\frac{y_0}{C}\right)^{1-m}$$

$$k' = r'(1-m)$$

![Fig. 3 The shape of growth function when $m < 0, \eta > 0, r < 0$](image)
IV. \( m < 1, \eta > 0, r > 0 \)

Using the expression (6) the shape for the growth function is given below.

![Graph showing growth function](image)

Fig. 4 The shape of growth function when \( m < 1, \eta > 0, r > 0 \)

CONCLUSION

- The difference between Fig. 1 and Fig. 2 is given by the feature of the growing. In Fig. 1 the growth is limited by the upper asymptote \( A \). The growth is limited by the genetic nature of living organism as well as by the carrying capacity of the environment. In Fig. 2 the growth is unbounded and the model is a little bit unrealistic but can be used when the aim is to rich some developing point for the forest.
- The third case (Fig. 3) shows that the growth will occur after some years. In the first years the grows is very slow and at some point the growth is faster and unlimited.
- The last case is the worst from practical point of view and it shows that a lot of trees will die due to some disease.

Upon the desire of the researcher the Bertalanffy-Richards growth model allow an easy choice for the parameters.

BIBLIOGRAPHY