COMPARATIVE ANALYSIS OF DEFORMATIONS IN GAUSS-KRÜGER AND STEREOGRAPHIC 1970 PROJECTIONS IN BELIŞ AREA, CLUJ COUNTY

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Abstract. Representing the earth's surface on a project plan is accompanied by deformations which depend on the type of chosen projection. It was analyzed how the chosen projection, Gauss-Krüger or Stereographic 1970, influence linear deformation in the Beliş area. It was found that in the studied area deformations in Gauss projection are quite big (30.48 cm / km) while the deformations in stereographic 1970 projection are lower (-7.88 cm / km) and negative.

Keywords: linear deformations, linear deformation module, Gauss-Krüger, Stereographic 1970, projection.

INTRODUCTION

When projecting the terrestrial surface onto a plane, deformations occur depending on the type of projection used. This article presents the influence of two conformal projections, Gauss-Krüger and Stereographic 1970, on distances.

The Gauss-Krüger projection is a transversal cylindrical projection with the cylinder being tangent to the axial meridian which has null linear distortions. The Gauss-Krüger projection is a conformal projection which preserves angles and distorts distances and areas. The distortions are positive and they increase as the distance to the axial meridian increases. The distortions have the allure of parallel lines with their axial meridian. The distortions of the marginal meridian have a value of 74 cm/km. (Palamariu, 2002, Munteanu, 2003).

Stereographic 1970 projection with single secant plane has the projection pole at north Fagaras, with geodesic coordinates of B=46° and L=25° and false rectangular coordinates of X = 500000 m and Y = 500000 m. It is a conformal projection that preserves angles in the projection plane, while the module of linear deformation in the projection pole is μ = 0,999750. This way distortions equal -25 cm/km in the projection pole. Linear deformations in the Stenographic 1970 projection have the allure of concentric circles centered in the projection pole. The circle of null distortions is placed at a distance of 201,718 km from projection pole. Within this circle linear distortions are negative and increase towards the center of the projection where they reach their maximum. Outside the circle of null distortion, deformations are positive and increase as the distance from this circle increases. In this projection distortions depend only on the distance from to the origin of the axes (Palamariu, 2002, Munteanu, 2003).

MATERIAL AND METHOD

The case study was performed in Beliş area, Cluj County. The measurements were done with the Leica TM 30 total station between the points PI, PII, PIII, PIV belonging to the surveying system of some hydro-electric objectives. The average distance between these points was then calculated based on the measurements.

Table 1

Measured distances

Point	Distance
PI-PII	199.84002
PI-PIII	282.55455
PI-PIV	235.28334
PII-PIII	103.76421
PII-PIV	158.79113
PIII-PIV	127.98909

The geodetic coordinates of the points were determined with a global positioning system.

Table 2

Geodetic coordinates of the points

Point	B	L
PI	46°40'11.90766"	23°03'24.95247"
PII	46°40'08.44527"	23°03'32.89501"
PIII	46°40'05.16186"	23°03'33.93331"
PIV	46°40'04.57737"	23°03'27.97257"

Geodetic coordinates were transformed in Gauss and Stereographic 1970 plane coordinates using the constant coefficients calculated by Struțu and Fălie (Munteanu, 2003).

For the Gauss projection the module of linear deformation was calculated by using two formulas (Munteanu, 2003) for validation:

$$\mu = 1 + \left[\frac{1}{2} \cdot cos^2 B \cdot (1 + \eta^2)\right] \cdot l^2 + \left[\frac{1}{24} \cdot cos^4 B \cdot (5 - t^2)\right] \cdot l^4 \quad (1)$$

Where,

$$\eta^2 = e'^2 \cdot cos^2 B$$
$$t^2 = tg^2 B$$

and

$$\mu = 1 + \frac{y^2}{2R_m^2} + \frac{y^4}{24R_m^4} \quad (2)$$

For the Gauss projection the module of linear deformation was calculated by using the formula (Palamariu, 2002):

$$\mu = 1 + \frac{s^2}{4R_0^{\ 2}} - \frac{1}{4000} \quad (3)$$

Relative deformations were calculated with the relation:

$$D = \mu - 1 \quad (4)$$

The distances reduced to the projection plan were then calculated by multiplying the measured distances with the deformation module.

RESULTS AND DISCUSSIONS

The Gauss-Krüger (table 3) and Stereographic 70 (table 4) coordinates were determined by using the constant coefficients method.

Table 3

Gauss-Krüger coordinates of the points

Point	X	Y
PI	5172705.609	4657403.341
PII	5172603.109	4657574.964
PIII	5172502.300	4657599.686
PIV	5172480.939	4657473.452

Table 4

Stenographic 1970 coordinates of the points

Point	X	Y
PI	576280.0075	351348.1902
PII	576168.992	351514.3392
PIII	576067.0996	351533.9183
PIV	576052.1631	351406.8135

The average linear deformation module for the area delimited by the four points was determined by applying the relations (1) and (2). The same result was obtained: μ_m = 1,00030483. The relative linear deformation was calculated for the area using the relation (4): D_m = 30,48291131 cm/km. Using the linear deformation module were calculated the distances reduced to the Gauss projection plane and then the differences between measured and reduced distances (table 5).

Table 5

The distances reduced to the Gauss projection plan and the differences between measured and reduced distances

Point	Measured distance	Distance reduced to Gauss projection plan	Difference between the measured distance and the distance reduced to the projection plan
PI-PII	199.8400	199.9009	-0.06092
PI-PIII	282.5546	282.6407	-0.08613
PI-PIV	235.2833	235.3551	-0.07172
PII-PIII	103.7642	103.7958	-0.03163
PII-PIV	158.7911	158.8395	-0.0484
PIII-PIV	127.9891	128.0281	-0.03901

The average deformation module for the area delimited by the four points in the Stereographic 70 projection was determined using relation (3), μ_m = 0,9999212. The relative linear deformation was also calculated: D_m = -7,883161688 cm/km. In the end, the distances reduced to the Stereographic 70 projection plane and the differences between measured and reduced distances (table 6) were calculated.

Table 6

The distances reduced to the Stereographic 70 projection plan and the differences between measured and reduced distances

Point	Measured distance	Distance reduced to Stereographic projection plan	Difference between the measured distance and the distance reduced to the projection plan
PI-PII	199.8400	199.8243	0.015754
PI-PIII	282.5546	282.5323	0.022274
PI-PIV	235.2833	235.2648	0.018548
PII-PIII	103.7642	103.7560	0.00818
PII-PIV	158.7911	158.7786	0.012518
PIII-PIV	127.9891	127.9790	0.01009

CONCLUSIONS

1. Deformation in the Gauss-Krüger projection is over unity, μ_m = 1,00030483, relative deformation beeing 30,48291131 cm / km, which shows that Beliş is located at the edge of the shaft 34. Deformation in Stereo 70 projections is subunitary, μ_m = 0,9999212, relative deformation being -7,883161688 cm/km, which means that Beliş is inside the circle of null distortions.

2. Distances reduced to the Gauss projection plane are greater than measured distances while the distances reduced to the Stereographic 70 projection plane are smaller than the measured ones.

3. When in the interest area geodetic-topographical work is carry out it is mandatory to take into account linear deformations due to projection system.

4. If the coordinates of a support network are given in a projection system, to stakeout in the interest area, the distances obtained from coordinates should be adjust with a correction equal with the invert of deformation module.

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