Bulletin UASVM Horticulture, 68(1)/2011
Print ISSN 1843-5254; Electronic ISSN 1843-5394

Aspects Regarding the Conversion of the Geographical Coordinates into Stereographical 1970 Coordinates

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Abstract. The paper treats the conversion of geographical coordinates into Stereographical 1970 coordinates. The transformation takes place in two distinct phases, which consist in transforming the geographic coordinates in rectangular coordinates on a tangent to the Krasovski ellipsoid in the projection pole and then the coordinates reduction of the tangent plan into the secant plan parallel to the tangent, located at a depth of 3.18 km. For checking the transformation, the inverse transformation is done, from rectangular coordinates in secant plan into rectangular coordinates in tangent plan and after into geographical coordinates. For the conversion is used the constant coefficients method with solving matrix.

Keywords: coordinates conversion, geographical coordinates, Stereographical 1970 coordinates

INTRODUCTION

The Stereographical 1970 projection is a perspective projection, oblique azimuth with unique secant plan, parallel with the tangent plan in the projection pole and perpendicular to the diameter that connects the main pole with the projection views. In terms of deformation is a conform projection that preserves angles in the project plan.

The geographical coordinates of the projection pole $Q_0(\varphi_0, \lambda_0)$,is chosen in the center area are:

$$\varphi_0 = 46^0 \text{ northern latitude}; \quad \lambda_0 = 25^0 \text{ eastern longitude} \tag{1}$$

The axis coordinate system has the origin in the projection pole Q_0(Xo = 0,000 m and Yo = 0,000 m), with the X axis disposed to north and the Y axis to east. In order to have positive coordinates throughout the country, the origin of the coordinates system was translated to the south and west by 500 km, obtaining the false coordinates Q_0(Xo = 500 000 m and Yo = 500 000 m).

In order to reduce the linear and areolar deformations was adopted a unique secant – 1970 plan, at a depth H = 3.18 km from the tangent plan.

After intersecting the sphere of average radius R_o = 6378.957 km, with the secant plan, resulted a circle of null deformations with the radius r_o = 201.718 km.

Inside the secant circle the deformations are negative, reaching at the pole projection the value of -0.25 m / km and zero outside the circle of null deformation (d> r_o), the relative linear deformation increases in positive value to value + 0.25 m / km at the distance d = 285 km from the center point of projection, respectively, to + 0.637 m / km at the distance d = 385 km.

The linear deformation module in the projection plan, for reducing the coordinates from the tangent plan into secant plan:

$$\mu = C = 1 - \frac{1}{4000} = 0.999750000 \quad (2)$$

For the reverse transformation of the stereographical plan coordinates into geographic coordinates we apply the coefficient of return to scale for the transition from the secant plan to tangent plan:

$$C' = 1/C = 1/0,999750000 = 1,000250063 \quad (3)$$

MATERIALS AND METHODS

The coordinates transformation from one system to another within the same datum it is also known under the name of conversion.

The coordinates conversion from geographic coordinates into Stereo 70 is achieved by means of constant coefficients which in the case of the Romanian territory were calculated for the center of projection Qo (φ_o = 46 °, λ_o = 25 °) and the Krasovski reference ellipsoid.

The operations to transform the coordinates (φ, λ) into STEREO (X, Y) coordinates is performed first on the tangent plan (x_{tg}, y_{tg}) and then on the secant plan (X_S Y_S), in the following sequence:

- Calculate the difference in geographical coordinates, expressed in sexagesimal seconds between the point of known coordinates and projection pole:

$$\Delta\varphi = (\varphi - \varphi_0)'', \quad \Delta\lambda = (\lambda - \lambda_0)'' \quad (4)$$

where:

φ – point latitude; λ – point longitude;

λ_0, φ_0 – latitude and longitude of the projection pole

- Calculate the variable coefficients

$$f = (\varphi - \varphi_0)''10^{-4}, \; l = (\lambda - \lambda_0)''10^{-4} \quad (5)$$

- Calculate the rectangular coordinates of the point, in tangent plan, with the origin of the coordinates system in the projection pole:

$$\begin{aligned} \Delta x'_{(1,1)} &= f^*_{(1,7)} A_{(7,4)} l^p_{(4,1)} \\ \Delta y'_{(1,1)} &= f^*_{(1,7)} B_{(7,4)} l^i_{(4,1)} \end{aligned} \quad (6)$$

where:

$$f_{(7,1)} = \begin{pmatrix} f^0 \\ f^1 \\ \ldots \\ f^6 \end{pmatrix}; \quad l^p_{(4,1)} = \begin{pmatrix} l^0 \\ l^2 \\ l^4 \\ l^6 \end{pmatrix}, \quad l^i_{(4,1)} = \begin{pmatrix} l^1 \\ l^3 \\ l^5 \\ l^7 \end{pmatrix} \quad (7)$$

A and B – constant coefficient matrix, calculated for the latitude of the projection pole (Moldoveanu 2002), [1]

$$A_{(7,4)} = \begin{pmatrix} a_{00} & a_{02} & a_{04} & a_{06} \\ a_{10} & a_{12} & a_{14} & a_{16} \\ \ldots & \ldots & \ldots & \ldots \\ a_{60} & a_{62} & a_{64} & a_{66} \end{pmatrix}; \quad B_{(7,4)} = \begin{pmatrix} b_{00} & b_{02} & b_{04} & b_{06} \\ b_{10} & b_{12} & b_{14} & b_{16} \\ \ldots & \ldots & \ldots & \ldots \\ b_{60} & b_{62} & b_{64} & b_{66} \end{pmatrix} \quad (8)$$

Observation: The lower indices from the relations (6), (7), (8) are vectors and matrix dimensions.

The stereographic coordinates reduction from tangent plan into secant plan:

$$\Delta x = C\Delta x'$$
$$\Delta y = C\Delta y' \tag{9}$$

- Calculate the “Stereographical 70” coordinates with the system origin translated:

$$X = X_0 + \Delta x$$
$$Y = Y_0 + \Delta y \tag{10}$$

To verify the transformation of geographical coordinates (φ, λ) coordinates into STEREO 70 is carried out similarly, the reverse transformation to obtain the latitude differences (Δφ) and longitude (Δλ) of the given point and the center of projection Qo (Xo, Yo). With the calculated differences in latitude and longitude coordinates and with the geographical coordinates of the projection pole it is calculated the geographical coordinates of the points of interest with the relation:

$$\varphi = \varphi_0 + \Delta\varphi$$
$$\lambda = \lambda_0 + \Delta\lambda \tag{11}$$

where:

φ_o, λ_o - geographical coordinates of the projection pole, presented in (1)

To calculate the differences in latitude and longitude to go through stages:

Calculate the stereographical coordinates difference, in secant plan, between the coordinates of the point of interest and the false coordinates of the projection pole, resulting the stereographic coordinates with the origin in the projection pole:

- Calculate the stereographic coordinates difference, in secant plan, between the coordinates of the point of interest and the false coordinates of the projection pole, resulting the stereographic coordinates with the origin in the projection pole:

$$\Delta X = (X - X_0)'', \quad \Delta Y = (Y - Y_0)'' \tag{12}$$

where:

X_0, Y_0 – false coordinates of the projection pole.

- Transformation of the stereographic coordinates from secant plan into tangent plan:

$$\Delta X_1 = C'\Delta X, \quad \Delta Y_1 = C'\Delta Y \tag{13}$$

- Calculate the variable coefficients:

$$F = \Delta X_1 10^{-5}, \, L = \Delta Y_1 10^{-5} \tag{14}$$

- Calculate the differences in latitude and longitude:

$$\Delta\varphi''_{(1,1)} = F^*_{(1,7)} C_{(7,4)} L^p_{(4,1)}$$
$$\Delta\lambda''_{(1,1)} = F^*_{(1,7)} D_{(7,4)} L^i_{(4,1)} \tag{15}$$

where:

$$f_{(7,1)} = \begin{pmatrix} f^0 \\ f^1 \\ \cdots \\ f^6 \end{pmatrix}; \quad l^p_{(4,1)} = \begin{pmatrix} l^0 \\ l^2 \\ l^4 \\ l^6 \end{pmatrix}, \quad l^i_{(4,1)} = \begin{pmatrix} l^1 \\ l^3 \\ l^5 \\ l^7 \end{pmatrix} \tag{16}$$

$$C_{(7,4)} = \begin{pmatrix} c_{00} & c_{02} & c_{04} & c_{06} \\ c_{10} & c_{12} & c_{14} & c_{16} \\ \cdots & \cdots & \cdots & \cdots \\ c_{60} & c_{62} & c_{64} & c_{66} \end{pmatrix}; \quad D_{(7,4)} = \begin{pmatrix} d_{00} & d_{02} & d_{04} & d_{06} \\ d_{10} & d_{12} & d_{14} & d_{16} \\ \cdots & \cdots & \cdots & \cdots \\ d_{60} & d_{62} & d_{64} & d_{66} \end{pmatrix} \tag{17}$$

Differences in latitude and longitude calculated by relation (15) are given in sexagesimal seconds, which will be transformed into degrees, minutes and seconds.

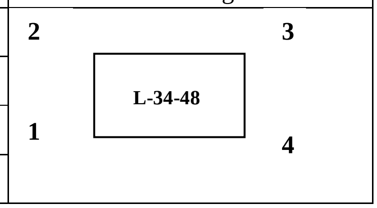
RESULTS AND DISCUSSION

For the case study were taken the geographic coordinates of the sheet plan L-34-48, which were transformed into coordinates "Stereo 70", applying the relations (4) - (10), and the conversion verification has been achieved with the relations (11) - (17).

Starting from the known geographic coordinates sheet plan at the scale 1:1 000 000 (L-34) and knowing how to obtain the sheet plans of scale 1:100 000, we determined the geographical coordinates of the sheet L-34-48 (C. Munteanu, 2003).

Tab. 1

Geographical coordinates values

Crt.	φ [o ‘ “]	λ [o ‘ “]	Drawing
1	$46^0 40'$	$23^0 30'$	
2	$47^0 00'$	$23^0 30'$	
3	$47^0 00'$	$24^0 00'$	
4	$46^0 40'$	$24^0 00'$	

Application of algorithms to convert geographic coordinates into stereographical 1970 and vice versa, will be presented for point 1, and for other points the calculation is similar.

Tab. 2

Calculus of the variable values

Latitude	φ [o ‘ “]	φ [“]	Longitude	λ [o ‘ “]	λ [“]
φ =	$46^0 40' 00''$	168000	λ =	$23^0 30' 00''$	2400
φ_0 =	$46^0 00' 00''$	165600	λ_0 =	$25^0 00' 00''$	0,2400
$\Delta\varphi = \varphi - \varphi_0 =$		2400	$\Delta\lambda = \lambda - \lambda_0 =$		-5400
$f = 10^{-4}\,\Delta\varphi =$		0,2400	$l = 10^{-4}\,\Delta\lambda =$		-0,5400

Column vectors of variable values:

$$f = \begin{pmatrix} f^0 \\ f^1 \\ f^2 \\ f^3 \\ f^4 \\ f^5 \\ f^6 \end{pmatrix} = \begin{pmatrix} 1{,}0000000 \\ 0{,}2400000 \\ 0{,}0576000 \\ 0{,}0138240 \\ 0{,}0033178 \\ 0{,}0007963 \\ 0{,}0001911 \end{pmatrix} \quad l^p = \begin{pmatrix} l^0 \\ l^2 \\ l^4 \\ l^6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0{,}2916000 \\ 0{,}0850306 \\ 0{,}0247949 \end{pmatrix}, \quad l^i = \begin{pmatrix} l^1 \\ l^3 \\ l^5 \\ l^7 \end{pmatrix} = \begin{pmatrix} -0{,}5400000 \\ -0{,}1574640 \\ -0{,}0459165 \\ -0{,}0133893 \end{pmatrix}$$

Constant coefficient matrix after (Moldoveanu, 2002):

$$A = \begin{pmatrix} 0 & 3752{,}145711 & 0{,}33593 & 0{,}00006 \\ 308758{,}958 & -99{,}92809 & -0{,}06434 & -0{,}00001 \\ 75{,}36099 & -6{,}67452 & 0{,}00038 & 0 \\ 60{,}21606 & -0{,}06838 & 0{,}00007 & 0 \\ -0{,}01495 & -0{,}00261 & 0 & 0 \\ 0{,}01410 & -0{,}00001 & 0 & 0 \\ -0{,}000001 & 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 215179{,}42084 & -23{,}21387 & -0{,}00897 & -5{,}1181E\text{-}07 \\ -10767{,}83826 & -1{,}92808 & 0{,}00056 & 0 \\ -128{,}66011 & 0{,}13593 & 0{,}00005 & 0 \\ -2{,}10685 & 0{,}00310 & 0 & 0 \\ -0{,}05003 & 0{,}00010 & 0 & 0 \\ -0{,}00051 & 0 & 0 & 0 \\ -0{,}00003 & 0 & 0 & 0 \end{pmatrix}$$

Applying equation (6) we obtain:

$$\Delta x'_{(1,1)} = f^{*}_{(1,7)} A_{(7,4)} l^{p}_{(4,1)} = 75194{,}3703\,\mathrm{m}$$

$$\Delta y'_{(1,1)} = f^{*}_{(1,7)} B_{(7,4)} l^{i}_{(4,1)} = -114793{,}6304\,\mathrm{m}$$

Reducing the stereographical coordinates in secant plan with the relation (9) we obtain:

$$\Delta x = C\Delta x' = 75175{,}5717\ m$$

$$\Delta y = C\Delta y' - 114764{,}932\ m$$

$$X = X_0 + \Delta x = 500000 + 75175{,}5717 = 575175{,}5717\ \mathrm{m}$$

$$Y = Y_0 + \Delta y = 500000 - 114764{,}932 = 385235{,}068\ \mathrm{m}$$

Performing the inverse transformation is obtained:

Tab. 3

Calculus of the variable values

Coordinate	Value	Coordinate	Value
x =	575175,57170	Y =	385235,06803
x_0=	500 000	Y_0=	500000
$\Delta X1=(X-X_0)C_1$=	75194,37033	$\Delta Y_1=(Y-Y_0)C_1$=	-114793,63043
$f= 10^{-5}\ \Delta X_1$=	0,751943703	$f= 10^{-5}\ \Delta Y_1$=	-1,147936304

Column vectors of variable values:

$$l^{p} = \begin{pmatrix} l^0 \\ l^2 \\ l^4 \\ l^6 \end{pmatrix} = \begin{pmatrix} 1 \\ 1{,}317757759 \\ 1{,}736485511 \\ 2{,}288267255 \end{pmatrix}, \quad l^{i} = \begin{pmatrix} l^1 \\ l^3 \\ l^5 \\ l^7 \end{pmatrix} = \begin{pmatrix} -1{,}147936304 \\ -1{,}512701971 \\ -1{,}99337476 \\ -2{,}626785055 \end{pmatrix}$$

$$f = \begin{pmatrix} f^0 \\ f^1 \\ f^2 \\ f^3 \\ f^4 \\ f^5 \\ f^6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0{,}751943703 \\ 0{,}565419333 \\ 0{,}425163507 \\ 0{,}319699022 \\ 0{,}240395667 \\ 0{,}180764008 \end{pmatrix}$$

Constant coefficient matrix after (Moldoveanu, 2002):

$$C = \begin{pmatrix} 0 & -26{,}24573021 & 0{,}00331221 & -4{,}3133E-07 \\ 3238{,}772428 & -0{,}620205898 & 0{,}000172568 & -5{,}518E-08 \\ -0{,}256027922 & -0{,}009981112 & 6{,}21731E-06 & 0 \\ -0{,}066216934 & -0{,}000188763 & 1{,}9998E-07 & 0 \\ 3{,}1693E-05 & -3{,}8399E-06 & 0 & 0 \\ 3{,}73701E-06 & -7{,}541E-08 & 0 & 0 \\ 3{,}458E-08 & 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 4647{,}28456 & -0{,}502080428 & 0{,}000112446 & 3{,}029E-08 \\ 75{,}31951041 & -0{,}02899936 & 1{,}05165E-05 & 0 \\ 1{,}506241284 & -0{,}001124457 & -6{,}3603E-07 & 0 \\ 0{,}02899936 & -3{,}50549E-05 & 0 & 0 \\ 0{,}000562228 & -1{,}06006E-06 & 0 & 0 \\ 1{,}05165E-05 & 0 & 0 & 0 \\ 2{,}1201E-07 & 0 & 0 & 0 \end{pmatrix}$$

Differences in latitude and longitude:

$$\Delta\varphi''_{(1,1)} = f^{*}_{(1,7)} C_{(7,4)} l^{p}_{(4,1)} = 2400{,}000002$$

$$\Delta\lambda''_{(1,1)} = f^{*}_{(1,7)} D_{(7,4)} l^{i}_{(4,1)} = -5400{,}000002$$

Calculating the geographical coordinates of point 1:

$$\varphi = \varphi_0 + \Delta\varphi = 46^0 40' 00'' + 0^0 40' 2'',26244\text{E-}06 = 46^0 40' 0'',000002$$

$$\lambda = \lambda_0 + \Delta\lambda = 23^0 30' 00'' - 1^0 30' 1'',50976\text{E-}06 = 23^0 29' 59'',99999849$$

After conversion of coordinates, it is noted that the difference between the transformed geographic coordinates from the initials is at the 6th number after the comma converted to geographical coordinates of the original is a comma at the seconds of arc.

Applying similarly the calculus algorithms for the other points of the trapezoid were obtained the values shown in Table 4.

Tab. 4

Transformed values of the points

Crt.	X [m]	Y [m]	φ [o ‘ “]	λ [o ‘ “]
2	612221,1477	385937,7231	$47^0 00' 00''$	$23^0 30' 00''$
3	611619,3977	423957,0889	$47^0 00' 00''$	$24^0 00' 00''$
4	574571,8008	423488,6651	$46^0 40' 00''$	$24^0 00' 00''$

To check the conversion of geographical coordinates into stereographical 1970 was used the Trapez program obtaining the following values:

Tab. 5

Differences between the initial and final values of the points

Crt.	Algorithm for computing		Trapez program		Differences	
	X [m]	Y [m]	X [m]	Y [m]	dX	dY
1	575175,57170	385235,06803	575175,572	385235,068	-0,0003	0,0000
2	612221,1477	385937,7231	612221,148	385937,723	-0,0003	0,0001
3	611619,3977	423957,0889	611619,398	423957,089	-0,0003	-0,0001
4	574571,8008	423488,6651	574571,801	423488,665	-0,0002	0,0001

CONCLUSIONS

After the realised survey we can outline the following conclusions:

- Conversion of geographical coordinates into stereographical 1970 it is used to calculate the inner corners of the dial of the plans and topographic maps;
- It is also used to calculate the coordinates of geodetic network points by conversion from geodetic coordinates B, L;
- Applying the calculation algorithms and the matrix solving method, the accuracy of the results is the tenth of a millimeter;
- In the case of conversion verification through inverse transformation, the difference between the calculated geographical coordinates and the initial ones is at the sixth decimal after the comma at the seconds of arc, which is the hundredth of a millimeter.

- Given the precision obtained in the conversion of coordinates and the verification of them, results that the set of constant coefficients [1] are correct.
- In the absence of calculus programs, coordinate conversion is done easily with Microsoft Excel.

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