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A New Definition of Singleton Probability for Continuous Probability Distributions

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Abstract. The elementary event should not appear. In other words, according to this definition none of the infinity of elementary events should appear. This is in disaccord with reality - elementary events do appear. The introduction of a new definition of singleton probability for continuous probability distributions solves both practical and theoretical problems. The new definition allows comparison of elementary event probabilities and solves a probability paradox. When throwing a dice, the event of appearance of the number seven has the probability zero. Even if the dice is rolled infinite times, the number seven definitely will not appear. In both cases, the probability of an event is zero, so in both cases the conclusion should be the same: the event will not happen. This seems to be a theoretical contradiction for elementary events, which follow a continuous distribution. The contradiction is discussed in the literature. The main idea of a Gaussian is that most observations relate to the middle, the average; the probability of deviations decreases more rapidly (exponentially) as deviating from the mean.

Keywords: probability density function, elementary event

INTRODUCTION

According to the classical definition, for a continuous distribution, the probability of a singleton is equal to zero. $P(a)=\int_a^a f(x)dx=0$, where $f(x)$ is a probability density function (Buiga *et al.*, 2003; Merce and Merce, 2009).

The elementary event should not appear. In other words, according to this definition none of the infinity of elementary events should appear. This is in disaccord with reality - elementary events do appear. When throwing a dice, the event of appearance of the number seven has the probability zero. Even if the dice is rolled infinite times, the number seven definitely will not appear.

In both cases, the probability of an event is zero so in both cases the conclusion should be the same: the event will not happen. This seems to be a theoretical contradiction for elementary events, which follow a continuous distribution. The contradiction is reported in the literature. The main idea of a Gaussian is that most observations relate to the middle, the average; the probabilities of deviations decrease more rapidly (exponentially) as deviating from the mean (Taleb, 2010). The probability theory tries to solve this contradiction arguing that this zero is a statistic zero. The author does not agree with this interpretationand proposes the use of a new definition of singleton probability for continuous probability distributions.

MATERIALS AND METHODS

Let's examine a few situations in which our intuition contradicts with the classical definition for elementary events. Let's assume that one have an appointment with a man for the first time. What can believe that it is most probable: the man has a height of 182 cm or a height of 214 cm? Most people will answer that the more probable is that the man has the height of 182 cm. The opinion of the author is that they are right, because 182 is closer to the average. According to the classical definition for singleton probability, assuming that the height of mature male population follows a Normal Distribution, the probability of meeting a man with a height of 182 cm is zero and the probability of meeting a man with a height of 214cm is zero. The probabilities are difficult to compare because both of them are zero. The simplest comparison may lead to the idea that the probability are equal, but any other conclusions may be derived because *N* times zero equals zero regardless of the value of *n*. In fact, it can be concluded that the first probability is five times the second probability and simultaneously the second probability is five time the first probability.

Generally known that meteorites collides Earth every year. What can believe that it is most probable: in the next century, a meteorite of three tones will collides with Earth or a meteorite of 31 tones? Most people will answer that the most probable is that a meteorite of three tones will collide with Earth in the next century. The answer is true, because 182 is closer to the average. According to the classical definition for singleton probability, assuming that the weight of meteorites colliding Earth follows an Exponential Distribution, the probability that a three tones meteorite will collide Earth in the next century is zero and the probability that a 31 tones meteorite will collide Earth is zero.Because both probabilities are zero, similar conclusions can be drawn such as those related to the previous example. In fact, may derive that the probability of the second event is a billion times more probable than the first one. In fact, this may be a very counterintuitive and faulty conclusion. In fact, a probability equal to absolute zero means that, according to the classic definition of a singleton probability, no meteorite will collide with Earth regardless of weight.

Let’s assume one spins a 1 m circumference wheel with all real numbers between 0m and 1 m marked ascending on it. What can believe that it is most probable to stop against an indicator: the number 0.2 or the number 0.6 ? Most people will answer that the probability of these two numbers are equal. None of them will say that the probability of apparition of number 0.2 is N times larger than the probability of apparition of number 0.6. According to the classical definition for singleton probability, assuming that the numbers follows an uniform continuous distribution, the probability of apparition of number 0.2 is zero and the probability of apparition of number 0.6 is zero. This means that it can be conclude that the probability of apparition of number 0.2 is equal to the probability of apparition of number 0.6 because both are zero. Another “correct” conclusion according to the classical definition is that the probability of apparition of number 0.2 is one million times larger than the probability of apparition of number 0.6.

RESULTS AND DISCUSSIONS

The paper gives a solution of the problems concerning the classical definition of singleton probability introducing a new definition:

$$P(a)=\lim_{h\to 0}\int_{a-h}^{a+h} f(x)dx$$

where, *h* represents the margin error.

According to the new definition the probability of a singleton will be:

$$P(a)=\lim_{h\to 0}\int_{a-h}^{a+h} f(x)dx=\lim_{h\to 0}\big(F(a+h)-F(a-h)\big)=$$
$$=\lim_{h\to 0}F(a+h)-\lim_{h\to 0}F(a-h)=F(a)-F(a)=0$$

where $f(x)$ the probability density is function and $F(x)$ is the cumulative distribution function. Between the probability density function $f(x)$ and the cumulative distribution function $F(x)$ exist the following identity: $F^{'}(x) = f(x)$. In other words, $F(x)$ is a primitive of the function $f(x)$.

According to the new definition, the probability of a singleton is not absolute zero but a statistic zero. This statistic zero makes a singleton highly improbable but not impossible.

The new definition allows the comparison of probability for two singletons, even if each singleton probability tends to zero. The two singleton probabilities will be compared taking into account the same error margin *h*.

$$\frac{P(a)}{P(b)} = \frac{\lim_{h\to 0}(F(a+h)-F(a-h))}{\lim_{h\to 0}(F(b+h)-F(b-h))} = \lim_{h\to 0}\frac{F(a+h)-F(a-h)}{F(b+h)-F(b-h)} =$$

$$= \lim_{h\to 0}\frac{\frac{F(a+h)-F(a-h)}{2\cdot h}}{\frac{F(b+h)-F(b-h)}{2\cdot h}} = \frac{\lim_{h\to 0}\frac{F(a+h)-F(a-h)}{2\cdot h}}{\lim_{h\to 0}\frac{F(b+h)-F(b-h)}{2\cdot h}} = \frac{f(a)}{f(b)}$$

It can therefore be concluded that:

$$\frac{P(a)}{P(b)} = \frac{f(a)}{f(b)} \quad (1)$$

This demonstration allows us to express the ratio between the probabilities of two singletons, which will be the ratio between the values of the probability density function.

Assuming that the distribution of adult men will be distributed according to a Normal distribution with parameters $\mu = 178$ and $\sigma = 7$, It can be calculated the ratio between probability to meet a men with a height of 182 cm and the probability to meet a men with a height of 214 cm.

The probability density function of the Normal distribution with parameters μ and σ will be:

$$f(x) = \frac{1}{\sigma\cdot\sqrt{2\cdot\Pi}}\cdot e^{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sigma\cdot\sqrt{2\cdot\Pi}}\cdot e^{-\frac{(x-\mu)^2}{2\cdot\sigma^2}} \qquad (2)$$

Taking into account the relations (1) and (2) with the parameters $\mu = 178$ and $\sigma = 7$, the ratio between probability to meet a men with a height of 182 cm and the probability to meet a men with a height of 214 cm.

$$\frac{P(182)}{P(214)} = \frac{\frac{1}{7\cdot\sqrt{2\cdot\Pi}}\cdot e^{-\frac{(182-178)^2}{2\cdot 49}}}{\frac{1}{7\cdot\sqrt{2\cdot\Pi}}\cdot e^{-\frac{(214-178)^2}{2\cdot 49}}} = \frac{e^{-\frac{(182-178)^2}{2\cdot 49}}}{e^{-\frac{(214-178)^2}{2\cdot 49}}} = 470346.29$$

The probability to meet a men with a height of 182 cm is 470346.29 times the probability to meet a men with a height of 214 cm.

Taking into account the relations (1) and (2), the singleton probability which is larger than any other singleton probability is the singleton which produces the maximum value for the probability density function of Normal distribution. This singleton will be $x_0 = \mu$.

Assuming that the distribution of meteorites, which collide with Earth, is an Exponential distribution with the parameter, $\lambda = 0.4$, it can be calculated the ratio between the probability of a 3 tones meteorite and the probability a 31 tones meteorite.

The probability density function of the Exponential distribution will be:

$$f(x)=\begin{cases}\lambda\cdot e^{-\lambda\cdot x} & x\geq 0\\ 0 & x<0\end{cases}$$

$$\frac{P(3)}{P(31)}=\frac{0,4\cdot e^{-0.4\cdot 3}}{0,4\cdot e^{-0.4\cdot 31}}=\frac{e^{-0.4\cdot 3}}{e^{-0.4\cdot 31}}=73130.44$$

The probability of a 3 tones meteorite colluding with Earth is 73130.44 times the probability a 31 tones meteorite colluding with Earth.

Assuming that the numbers resulting from the spinning wheel will follows a Continuous Uniform distribution it can be calculated the ratio between the probabilities of singletons 0.2 and 0.6 using the relation (1). The general case for this type of distribution is:

$$\begin{cases}0 & x<a\\ \frac{1}{b-a} & x\in[a;b]\\ 0 & x>b\end{cases}$$

The ratio between the probabilities of apparition of number 0.2 and the probability of apparition of number 0.6 will be:

$$\frac{P(0.2)}{P(0.6)}=\frac{\frac{1}{1-0}}{\frac{1}{1-0}}=\frac{1}{1}=1$$

The probability of apparition of number 0.2 equals the probability of apparition of number 0.6.

Assuming that the population follows a continous distribution, it is possible to compare probabilities of elementary events. The new definition can be used to demonstrate the Maximum Likelihood Method taking into account that different distributions reflect the same population.

CONCLUSION

The new definition solves a probability paradox. Using the new definition, the probability of singleton is not an absolute zero, but a relative zero, which may be named a statistic zero. The new definition may affect research in various sciences and it can be used to demonstrate the Maximum Likelihood Method taking into account that different distributions reflect the same population.

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